Leptogenesis and CP Violation in Neutrino Oscillations

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Assuming the seesaw and leptogenesis mechanisms, I give some general remarks on possible connection or disconnection between CP violation in heavy Majorana neutrino decays and that in neutrino oscillations. A simple but predictive ansatz is proposed to simultaneously interpret the observed baryon asymmetry of the universe and oscillations of solar and atmospheric neutrinos.

The density of baryons compared to that of photons in the universe is extremely small: $\eta \equiv n_{\rm B}/n_\gamma = (2.6 - 6.3) \times 10^{-10}$, extracted from the Big-Bang nucleosynthesis [1]. This tiny quantity is related to the observed matter-antimatter or baryon-antibaryon asymmetry of the universe,

$$Y_{\rm B} \equiv \frac{n_{\rm B}}{\mathbf{s}} \approx \frac{\eta}{7.04} = (3.7 - 8.9) \times 10^{-11}, \qquad (1)$$

where $\mathbf{s}$ denotes the entropy density. In order to produce a net baryon asymmetry in the standard Big-Bang model, three Sakharov necessary conditions must be satisfied [2]: (a) baryon number nonconservation, (b) C and CP violation, and (c) a departure from thermal equilibrium. Among several interesting and viable baryogenesis scenarios proposed in the literature, Fukugita and Yanagida's leptogenesis mechanism [3] has attracted a lot of attention – due partly to the fact that neutrino physics is entering a flourishing era.

Indeed the Super-K [4] and SNO [5] data have provided very convincing evidence that neutrinos are massive and lepton flavors are mixed – a kind of new physics which can only be understood beyond the standard electroweak model. A simple extension of the standard model is to include one right-handed neutrino in each of three lepton families, while the Lagrangian of electroweak interactions keeps invariant under the ${\rm SU(2)_L \times U(1)_Y}$ gauge transformation. In this case, the Yukawa interactions are described by

$$-\mathcal{L}_{\rm Y} = \bar{l}_{\rm L}\tilde{\phi}Y_l e_{\rm R} + \bar{l}_{\rm L}\phi Y_\nu \nu_{\rm R} + \frac{1}{2}\overline{\nu^{\rm c}_{\rm R}} M_{\rm R}\nu_{\rm R} + {\rm h.c}, \qquad (2)$$

where $l_{\rm L}$ denotes the left-handed lepton doublet, $e_{\rm R}$ and $\nu_{\rm R}$ stand respectively for the right-handed charged lepton and Majorana neutrino singlets, and ϕ is the standard-model Higgs doublet. The lepton number violation induced by the third term of $\mathcal{L}_{\rm Y}$ allows decays of the heavy right-handed Majorana neutrinos N_i (for $i = 1, 2, 3$) to occur:

$$N_i \rightarrow l + \phi^\dagger \quad {\rm vs} \quad N_i \rightarrow l^{\rm c} + \phi \; . \qquad (3)$$

As each decay can happen both at the tree level and at the one-loop level (via the self-energy and vertex corrections), the interference between tree-level and one-loop decay amplitudes may give rise to a CP-violating asymmetry ε_i between the two CP-conjugated processes in Eq. (3) [3]. If the masses of three heavy Majorana neutrinos N_i are hierarchical ($M_1 \ll M_2 \ll M_3$) and the interactions of N_1 are in thermal equilibrium when N_2 and N_3 decay, the asymmetries produced by N_2 and N_3 can be erased before N_1 decays. The CP asymmetry ε_1 produced by the out-of-equilibrium decay of N_1 survives, and it results in a lepton-antilepton asymmetry $Y_{\rm L} \equiv n_{\rm L}/\mathbf{s} = \varepsilon_1 d/g_*$, where $g_* \sim 100$ is an effective number characterizing the relativistic degrees of freedom which contribute to the entropy $\mathbf{s}$, and d accounts for the dilution effects induced by the lepton-number-violating wash-out processes. Finally the lepton asymmetry $Y_{\rm L}$ is converted into a net baryon asymmetry $Y_{\rm B}$ through the $({\rm B+L})$-violating sphaleron processes [6]:

$$Y_{\rm B} = \frac{c}{c-1} Y_{\rm L} = \frac{c}{c-1} \cdot \frac{d}{g_*} \varepsilon_1 \; , \qquad (4)$$

where $c = (8{\rm N}_f + 4{\rm N}_\phi)/(22{\rm N}_f + 13{\rm N}_\phi)$ with ${\rm N}_f$ being the number of fermion families and ${\rm N}_\phi$ being the number of Higgs doublets. Taking $\mathrm{N}_f = 3$ and $\mathrm{N}_\phi = 1$ for example, we obtain $c \approx 1/3$.

After spontaneous symmetry breaking, we get the charged lepton mass matrix $M_l = Y_l \langle \phi \rangle$ and the Dirac neutrino mass matrix $M_{\rm D} = Y_\nu \langle \phi \rangle$ from $\mathcal{L}_{\rm Y}$, in addition to the right-handed Majorana neutrino mass matrix $M_{\rm R}$. The scale of M_l and $M_{\rm D}$ is characterized by the gauge symmetry breaking scale $v \equiv \langle \phi \rangle \approx 175$ GeV, but that of $M_{\rm R}$ may be much higher than v, because right-handed neutrinos are $\mathrm{SU(2)_L}$ singlets and their mass term is not subject to the electroweak symmetry breaking. As a consequence, the light (and essentially left-handed) neutrino mass matrix M_ν can be obtained via the seesaw mechanism [7]:

$$M_\nu \approx - M_{\rm D} M^{-1}_{\rm R} M^{\rm T}_{\rm D} \; . \tag{5}$$

Note that lepton flavor mixing at low energy scales stems from a nontrivial mismatch between the diagonalizations of M_ν and M_l, while the baryon asymmetry at high energy scales depends on complex $M_{\rm D}$ and $M_{\rm R}$ in the leptogenesis scenario. To see these points more clearly, let us diagonalize three of the four lepton mass matrices: $U^\dagger_l M_l \tilde{U}_l = {\rm Diag}\{m_e, m_\mu, m_\tau\}$ and $U^\dagger_\nu M_\nu \tilde{U}^*_\nu = {\rm Diag}\{m_1, m_2, m_3\}$ as well as $U^\dagger_{\rm R} M_{\rm R} \tilde{U}^*_{\rm R} = {\rm Diag}\{M_1, M_2, M_3\}$. Then the lepton flavor mixing matrix at low energy scales is given by $V = U^\dagger_l U_\nu$, and the leptonic CP violation in neutrino oscillations is measured by the Jarlskog parameter $\mathcal{J}$ defined through

$${\rm Im} \left(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \right) = \mathcal{J} \sum_{\gamma, k} \left(\epsilon_{\alpha\beta\gamma} \cdot \epsilon_{ijk} \right) , \tag{6}$$

where (α, β) run over (e, μ, τ) and (i, j) run over $(1, 2, 3)$. On the other hand, the CP asymmetry ε_1 between $N_1 \rightarrow l + \phi^\dagger$ and $N_1 \rightarrow l^{\rm c} + \phi$ decays at high energy scales can be given as [8]

$$\varepsilon_1 = - \frac{3}{16\pi v^2} \cdot \frac{M_1}{\left[U^{\rm T}_{\rm R} M^\dagger_{\rm D} M_{\rm D} U^*_{\rm R} \right]_{11}} \cdot \sum^3_{j=2} \frac{{\rm Im} \left(\left[U^{\rm T}_{\rm R} M^\dagger_{\rm D} M_{\rm D} U^*_{\rm R} \right]_{1j} \right)^2}{M_j} \; , \tag{7}$$

where the mass hierarchy of three heavy Majorana neutrinos ($M_1 \ll M_2 \ll M_3$) has been assumed. We see that $\mathcal{J}$ depends on U_l and U_ν or equivalently on M_l and M_ν, while ε_1 depends on $M_{\rm D}$ and $M_{\rm R}$ (or $U_{\rm R}$ and M_i). The only possible relationship between $\mathcal{J}$ and ε_1 is due to the seesaw mechanism in Eq. (5), which links M_ν to $M_{\rm D}$ and $M_{\rm R}$. Therefore one can conclude that there is no direct connection between CP violation in heavy Majorana neutrino decays (ε_1) and that in neutrino oscillations ($\mathcal{J}$). Such a general conclusion was first drawn by Buchmüller and Plümacher [9]. Recently a few other authors have carried out some more delicate analyses and reached the same conclusion [10].

Depending on the specific flavor basis that we choose in model building, $\mathcal{J}$ and ε_1 can either be completely disconnected or maximally connected. To illustrate, let us consider two extreme cases:

(1) In the basis where $U_\nu = \mathbf{1}$ holds (i.e., $V = U^\dagger_l$ – lepton flavor mixing and CP violation in neutrino oscillations arise solely from the charged lepton sector [11]), we find that ε_1 has nothing to do with V or $\mathcal{J}$. In this special case, less fine-tuning is expected in building a phenomenological model which can simultaneously interpret the baryon asymmetry of the universe and lepton flavor mixing at low energy scales.

(2) In the basis where both $U_{\rm R} = \mathbf{1}$ and $U_l = \mathbf{1}$ hold (i.e., $V = U_\nu$ – lepton flavor mixing and CP violation in neutrino oscillations arise solely from the neutrino sector), we find that ε_1 can indirectly be linked to V and $\mathcal{J}$ through the seesaw relation in Eq. (5). It is worth remarking that the correlation between high- and low-energy observable quantities requires quite nontrivial attempts in model building. In particular, the textures of $M_{\rm R}$ and $M_{\rm D}$ have to be carefully chosen or fine-tuned to guarantee acceptable agreement between the model predictions and the observational or experimental data.

In the second part of this talk, I propose a phenomenological ansatz to simultaneously interpret the observed baryon asymmetry of the universe and neutrino oscillations. Our simple ansatz [8] is essentially a non-SO(10) modification of the Buchmüller-Wyler ansatz [12], but it has more powerful predictability and its predictions are in better agreement with current data.

First of all, we assume M_l and $M_{\rm D}$ to be symmetric matrices, just like $M_{\rm R}$. Second, we assume that the (1,1), (1,3) and (3,1) elements of M_l, $M_{\rm D}$ and $M_{\rm R}$ are all vanishing in a specific flavor basis, like a phenomenologically-favored texture of quark mass matrices $M_{\rm u}$ and $M_{\rm d}$ [13]. Third, we assume that the non-zero elements of $M_{\rm D}$ and M_l can be expanded in terms of the Wolfenstein parameter $\lambda \approx 0.22$. To be explicit, we conjecture that $M_{\rm D}$ and M_l have the following patterns:

$$\frac{M_{\rm D}}{m_0} = \begin{bmatrix} 0 & \hat{\lambda}^3 & 0 \\ \hat{\lambda}^3 & x\hat{\lambda}^2 & \hat{\lambda}^2 \\ 0 & \hat{\lambda}^2 & e^{i\zeta} \end{bmatrix}, \frac{M_l}{m_\tau} = \begin{bmatrix} 0 & \lambda^4 & 0 \\ \lambda^4 & y\lambda^2 & \lambda^3 \\ 0 & \lambda^3 & 1 \end{bmatrix}$$

with $m_0 \approx v$, $\hat{\lambda} \equiv \lambda e^{i\omega}$ and (x, y) being real and positive coefficients of $\mathcal{O}(1)$. It is easy to check that three mass eigenvalues of $M_{\rm D}$ have the hierarchy $\lambda^4 : \lambda^2 : 1$, and those of M_l have the hierarchy compatible with our experimental data on m_e, m_μ and m_τ. As the (1,1), (1,3) and (3,1) elements of both $M_{\rm R}$ and $M_{\rm D}$ are vanishing, M_ν must have the same texture zeros via the seesaw relation in Eq. (5) [14]. To generate a sufficiently large mixing angle in the ν_μ-ν_τ sector, (2,3), (3,2) and (3,3) elements of M_ν should be comparable in magnitude. This requirement is actually strong enough to constrain the texture of $M_{\rm R}$ in a quite unique way [12]. For our purpose, we obtain

$$\frac{M_{\rm R}}{M_0} = \begin{bmatrix} 0 & \lambda^5 & 0 \\ \lambda^5 & z\lambda^4 & \lambda^4 \\ 0 & \lambda^4 & 1 \end{bmatrix}, \frac{M_\nu}{m'_0} = \begin{bmatrix} 0 & \hat{\lambda} & 0 \\ \hat{\lambda} & z' & 1 \\ 0 & 1 & e^{i2\varphi} \end{bmatrix},$$

where $M_0 \gg v$, $m'_0 = v^2/M_0$, z is a real and positive coefficient of $\mathcal{O}(1)$, $z' \equiv 2x - ze^{i\omega}$ with $|z'| \sim \mathcal{O}(1)$, and $2\varphi \equiv 2\zeta - 5\omega$. Note that an overall phase factor $e^{i(5\omega-\pi)}$ has been omitted from the right-hand side of M_ν, since it has no contribution to lepton flavor mixing and CP violation. To generate a large mixing angle in the ν_e-ν_μ sector, the condition $|z' e^{i2\varphi} - 1| \equiv \delta \sim \mathcal{O}(\lambda)$ must be satisfied. There exists an interesting parameter space, in which [8] $x = 1/\sqrt{2}$, $z = 1 + \sqrt{2}\lambda$ and $\zeta = -\omega = \pi/4$. One may check that $\delta = \sqrt{2}\lambda$ holds in this parameter space.

As mentioned above, complex M_ν can be diagonalized by a unitary matrix U_ν, On the other hand, the strong hierarchy of real M_l under consideration implies that its contribution to lepton flavor mixing is small and negligible. Then we arrive at $V \approx U_\nu$, which links the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$. Current data on solar and atmospheric neutrino oscillations suggest $|V_{e3}| \ll 1$, $|V_{e1}| \sim |V_{e2}|$ and $|V_{\mu 3}| \sim |V_{\tau 3}|$. Hence a parametrization of V needs two big mixing angles (θ_x and θ_y) and one small mixing angle (θ_z), in addition to a few complex phases. Following Ref. [8], we obtain

$$\tan 2\theta_x \approx 2\sqrt{2}\frac{\lambda}{\delta} \ , \ \ \tan 2\theta_y \approx \frac{2}{\delta} \ , \ \ \tan 2\theta_z \approx \frac{\lambda}{\sqrt{2}} \ .$$

Taking $\delta = \sqrt{2}\lambda$, we explicitly obtain $\theta_x \approx 31.7^\circ$, $\theta_y \approx 40.6^\circ$ and $\theta_z \approx 4.4^\circ$, favored by current experimental data. In addition, the masses of three light neutrinos are given by

$$\frac{m_1}{m_3} \approx \frac{\lambda \tan\theta_x}{2\sqrt{2}} \ , \ \ \frac{m_2}{m_3} \approx \frac{\lambda \cot\theta_x}{2\sqrt{2}} \ , \ \ m_3 \approx \frac{2m_0^2}{M_0} \ .$$

We see that a normal neutrino mass hierarchy $m_1 : m_2 : m_3 \sim \lambda : \lambda : 1$ shows up. Then the absolute value of m_3 can be determined from the observed mass-squared difference of atmospheric neutrino oscillations $\Delta m^2_{\rm atm} \equiv |m_3^2 - m_2^2| \approx m_3^2$. Using $\Delta m^2_{\rm atm} = (1.6 - 3.9) \times 10^{-3}$ eV^2 [15], we obtain $m_3 \approx \sqrt{\Delta m^2_{\rm atm}} \approx (4.0 - 6.2) \times 10^{-2}$ eV. Given $m_0 \approx v$ for the Dirac neutrino mass matrix $M_{\rm D}$, the mass scale of three heavy Majorana neutrinos turns out to be $M_0 \approx 2v^2/m_3 \approx (4.9 - 7.6) \times 10^{14}$ GeV, which is quite close to the scale of grand unified theories $\Lambda_{\rm GUT} \sim 10^{16}$ GeV.

Note that our ansatz predicts a very small value for the effective mass term of the neutrinoless double-β decay: $\langle m \rangle_{ee} \approx \lambda^2 m_3/8 \approx (2.4 - 3.8) \times 10^{-4}$ eV, which is much lower than the present experimental upper bound ($\langle m \rangle_{ee} < 0.35$ eV at the 90% C.L.[16]) and seems hopeless to be detected in practice. We also obtain the Jarlskog parameter, which measures CP and T violation in neutrino oscillations, as $\mathcal{J} \approx \lambda/(4\sqrt{10}) \approx 2\%$. Leptonic CP violation at the percent level could be measured in the future at neutrino factories.

The symmetric mass matrix $M_{\rm R}$ can be diagonalized by a unitary matrix $U_{\rm R}$, as pointed out above. To leading order, we find $M_1 \approx \lambda^6 M_0/z$, $M_2 \approx z\lambda^4 M_0$ and $M_3 \approx M_0$ as well as $U_{\rm R11} \approx i$, $U_{\rm R22} \approx U_{\rm R33} \approx 1$, $U_{\rm R12} \approx iU_{\rm R21} \approx \lambda/z$, $U_{\rm R13} \approx$

0, $U_{\rm R31} \approx i\lambda^5/z$, and $U_{\rm R23} \approx -U_{\rm R32} \approx \lambda^4$. One can see that the masses of three heavy Majorana neutrinos perform a strong hierarchy. Note that $\{M_1, M_2, M_3\} \approx \{5.2\times 10^{10}, 1.8\times 10^{12}, 6.0\times 10^{14}\}$ GeV, if we typically take $M_0 = 6.0 \times 10^{14}$ GeV and $z = 1+\sqrt{2}\lambda$.

A CP-violating asymmetry (ε_1) may result from the interference between tree-level and one-loop amplitudes of the decay of the *lightest* heavy Majorana neutrino N_1, as already presented in Eq. (7). In our ansatz, we can explicitly obtain

$$\varepsilon_1 \approx -\frac{3\lambda^6}{16\pi}\left[\sin 2(2\omega-\zeta) - 2x(1+x^2)\sin\omega \right.$$
$$\left. +x^2 z\sin 2\omega\right]\left[z\left(1+x^2+z^2-2xz\cos\omega\right)\right]^{-1} .$$

For illustration, we adopt the specific parameter space chosen above to evaluate the size of ε_1. The result is $\varepsilon_1 \approx -5.2\times 10^{-6}$. To translate this CP asymmetry into the lepton asymmetry $Y_{\rm L}$ and the baryon asymmetry $Y_{\rm B}$, it is necessary to calculate the dilution factor d appearing in Eq. (4). Note that d depends closely on the following quantity: $K_{\rm R} \equiv [U^{\rm T} M^\dagger_{\rm D} M_{\rm D} U^*]_{11} M_{\rm Pl}/(8\pi v^2 \cdot 1.66\sqrt{g_*}M_1)$ with $M_{\rm Pl} \approx 1.22\times 10^{19}$ GeV, which characterizes the out-of-equilibrium decay rate of N_1. Making use of the typical inputs taken above, we arrive at $K_{\rm R} \approx 73$. The dilution factor d can then be calculated with the help of an approximate parametrization [17] obtained from integrating the Boltzmann equations (for the range $10 \leq K_{\rm R} \leq 10^6$): $d \approx 0.3/[K_{\rm R}(\ln K_{\rm R})^{0.6}] \approx 1.7\times 10^{-3}$. Finally we get a very instructive prediction for the baryon asymmetry of the universe from Eq. (4): $Y_{\rm B} \approx 4.7\times 10^{-11}$. One can see that this result is consistent quite well with the observational value of $Y_{\rm B}$ quoted in Eq. (1).

One may go beyond the typical parameter space taken in this talk to make a delicate analysis of all measurables or observables. It is remarkable that we can quantitatively interpret both the baryon asymmetry of the universe and the small mass-squared differences and large mixing factors of solar and atmospheric neutrino oscillations. In this sense, our ansatz is a *complete* phenomenological ansatz favored by current experimental and observational data, although it has not been incorporated into a convincing theoretical model.

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